Stability of a class of neutral vacuum bubbles

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A model that gives rise to vacuum bubbles is considered where the domain wall field interacts with another real scalar field, resulting in the formation of domain ribbons within the host domain wall. Ribbon-antiribbon annihilations produce elementary bosons whose mass inside the wall is different from the mass in vacuum. Two cases are considered, where the bosons get trapped either within the bubble wall or the bosons get trapped within the vacuum enclosed by the bubble. The bosonic (meta)stabilization effect on the bubble is examined in each case. It is found that when the bosons become trapped within the bubble wall, the stabilization mechanism lasts for only a limited amount of time, and then the bubble undergoes unchecked collapse. However, when the bosons become trapped within the bubble's interior volume, the bubble can be long-lived, provided that it has a sufficiently thin wall.

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I. INTRODUCTION

The stabilizing effects of fermions on vacuum bubbles and other nontopological solitons have been studied in a number of interesting field theoretical scenarios[1]-[9]. In addition, models have been examined where such objects may be stabilized by bosons which can get trapped within the defect, due to a mass contrast between interior and exterior regions[10]-[14]. Presently, we consider models for possible bosonic stabilizing effects, where elementary bosons are produced by annihilations of defects ("domain ribbons") residing within the host defect (domain wall). This is similar in nature to a recent study investigating the possibility of a boson gas (meta)stabilization of cosmic string loops[15].

The domain wall is described by a real scalar field χ which interpolates between true vacuum states $\chi = \pm\eta$, but the field χ also interacts with a second scalar field ϕ, as described in Ref.[16]. By the Witten mechanism[17], there is a certain range of model parameters allowing the scalar ϕ to settle into nonzero vacuum states within the core of the wall, taking on values where $\phi = \pm\phi_0$, thereby breaking a discrete Z_2 symmetry associated with the field ϕ within the wall. However, the $+\phi_0$ and $-\phi_0$ vacuum domains will be uncorrelated beyond some coherence length ξ, and a $+\phi_0$ domain and an adjacent $-\phi_0$ domain will be separated by region where $\phi = 0$, which locates the center of a domain "ribbon" or "antiribbon" described by ϕ_R and $\phi_{\bar{R}}$. So, a ribbon or antiribbon is a topological domain wall section trapped within the host domain wall formed by the field χ. A single ribbon solution interpolates between

the vacuum states $\pm\phi_0$, and two adjacent ribbons are separated by an antiribbon, and vice versa.

The ribbons and antiribbons are dynamical objects which can interact with each other and themselves, undergoing annihilations and formations of ribbon loops through fission and fusion processes. A ribbon loop is surrounded by a $\pm\phi_0$ domain and encloses a $\mp\phi_0$ domain. A section of ribbon (R) and a section of antiribbon ($\bar{\mathrm{R}}$) can undergo annihilation, resulting in the formation of elementary φ bosons, which are the perturbative particle excitations of the ϕ field. The φ bosons have a mass $m_{\rm in}$ inside the domain wall and a mass m outside the wall in a true vacuum. If there is a high mass contrast between $m_{\rm in}$ and m, then φ bosons will tend to either become trapped within the wall ($m_{\rm in} \ll m$) or be expelled out of the wall ($m_{\rm in} \gg m$). A formation of χ vacuum bubbles can result, due to wall-wall interactions or self-intersecting trajectories. Also, if the Z_2 symmetry with degenerate vacuum states $\chi = \pm\eta$ is biased with different probabilities of forming different domains becoming unequal[18], or if the Z_2 symmetry is approximate with a negligible difference in vacuum energies[19], a network of bounded domain wall surfaces may result, leading to vacuum bubbles. The φ particles which either get trapped within the bubble wall or the interior volume of the bubble can then have a (meta)stabilizing effect. Although the issue of long term bubble stability is difficult to address with confidence, the shorter term bosonic stabilizing effect can be analyzed for both cases $m_{\rm in} \ll m$ and $m_{\rm in} \gg m$.

The basic domain ribbon model is described in Sec. II. Bosonic stabilization is then analyzed in Sec.III for the case $m_{\rm in} \ll m$ and in Sec. IV for the case $m_{\rm in} \gg m$. Equilibrium bubble radii and bubble masses are obtained for each case, and the possibility of long term stability is considered. It is concluded that for the case where φ bosons become trapped within the bubble wall ($m_{\rm in} \ll m$), the bosonic stabilization mechanism is initially effective, with a slow rate of leakage of φ gas due to a small number of bosons with energies $E \geq m$ escaping the wall. However, the gas temperature subsequently rises, and the stability mechanism eventually becomes ineffective, with much of the φ boson gas escaping the bubble wall. On the other hand, for the case where the φ gas is trapped within the bubble's interior volume ($m_{\rm in} \gg m$), there is a possibility for long term stability, provided that the bubble wall is thin enough. A brief summary is presented in Sec. V.

II. DOMAIN RIBBON MODEL

The Lagrangian for the system of two interacting scalar fields χ and ϕ is[16]

$$\mathcal{L} = \frac{1}{2}\partial^\mu\chi\partial_\mu\chi + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\chi,\phi) \tag{1}$$

where the potential is

$$V(\chi,\phi) = \frac{1}{4}\lambda\left(\chi^2-\eta^2\right)^2 + \frac{1}{2}f\left(\chi^2-\eta^2\right)\phi^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4}g\phi^4 \tag{2}$$

The stable vacuum states of this system are located by $\chi = \pm\eta$, $\phi = 0$ The parameters λ, f, η, m, and g are taken to be real-valued and positive. The field equations obtained from $\mathcal{L}$ are

$$\nabla_\mu \partial^\mu \chi + \left[\lambda\left(\chi^2 - \eta^2\right) + f\phi^2\right]\chi = 0 \tag{3}$$

$$\nabla_\mu \partial^\mu \phi + \left[f\left(\chi^2 - \eta^2\right) + m^2 + g\phi^2\right]\phi = 0 \tag{4}$$

We notice that for $\phi = 0$ the field equation for χ admits a domain wall solution describing a planar domain wall centered on the $y - z$ plane:

$$\chi(x) = \eta \tanh\left(\frac{x}{w}\right), \qquad w = \frac{1}{\eta}\sqrt{\frac{2}{\lambda}} \tag{5}$$

where w is the thickness of the wall. Now, if the field ϕ does not vanish identically, the Witten mechanism[17] allows a ϕ condensate to form within the wall taking values $\phi = \pm\phi_0$, where

$$\phi_0 = \left[\left(f\eta^2 - m^2\right)/g\right]^{\frac{1}{2}} > 0 \tag{6}$$

For mathematical simplicity and approximation purposes we take the wall to be a slab of thickness w, with

$$|\chi| \approx \left\{\begin{array}{l} 0,\ |x| \le \frac{1}{2}w \\ \eta,\ |x| > \frac{1}{2}w \end{array}\right\}, \qquad \phi \approx \left\{\begin{array}{cc} \phi(y,z,t), & |x| \le \frac{1}{2}w \\ 0, & |x| > \frac{1}{2}w \end{array}\right\}. \tag{7}$$

Then (4), with the help of (7), allows us to express the equation of motion for ϕ inside the domain wall by, approximately,

$$\partial_0^2\phi - \left(\partial_y^2\phi + \partial_z^2\phi\right) + g\phi\left(\phi^2 - \phi_0^2\right) = 0 \tag{8}$$

Equation (8) admits the static solution

$$\phi_R(z) = \phi_0 \tanh\left(\frac{z - z_0}{w_R}\right), \qquad w_R = \frac{1}{\phi_0}\sqrt{\frac{2}{g}} \tag{9}$$

describing a *domain ribbon* of width w_R embedded within the wall, lying along the y direction with $\phi_R \to \pm\phi_0$ as $z \to \pm\infty$. The ribbon (R) thus separates $\pm\phi_0$ domains, and the antiribbon ($\bar{\mathrm{R}}$) solution is given by $\phi_{\bar{R}}(z) = -\phi_R(z)$. We picture the ribbon as a section of domain wall, lying on the y axis, with thickness w_R in the z direction and thickness w in the x direction.

In the domain wall background described by (7), we see from (2) and (6) that $\phi = 0$ is not the lowest energy state inside the χ wall, but rather $\phi = \pm\phi_0$ is. Therefore, a ϕ field forms a condensate within the wall, tending to settle into either $\phi = +\phi_0$ or $\phi = -\phi_0$ domains, but these domains will be uncorrelated beyond some coherence length $\xi \gtrsim w_R \sim 1/(\sqrt{g}\phi_0)$. Two different adjacent domains must be separated by a ribbon or antiribbon located where $\phi = 0$. More general solutions of (8) would include infinite ribbons, which need not be straight or static, and closed (anti)ribbon loops[16]. These ribbon configurations therefore

resemble cosmic strings. A closed loop is surrounded by a $\pm\phi_0$ domain, and encloses a $\mp\phi_0$ domain. Smaller loops can fuse together to form larger loops, and larger loops can fission by self-intersection to form smaller loops. These processes involve R-$\bar{\text{R}}$ annihilations wherever the loops intersect, resulting in the release of φ boson radiation, where the φ boson is the elementary particle excitation of the ϕ field inside the χ wall, i.e., $\phi = \phi_0 + \varphi$, for example. For our model, we take the φ particles to be much less massive than loops, which have masses $M_{\rm loop} = \mu_R L$, where μ_R is the linear energy density of a ribbon (tension), and L is the loop length. Therefore, we assume R-$\bar{\text{R}}$ annihilations resulting in φ production to be much more likely than loop creation due to φ scattering.

The φ particle mass inside the χ wall is denoted by $m_{\rm in}$, while the ϕ particle mass outside the wall is denoted by m. There are two possibilities we wish to consider.

(1) $m_{\rm in} \ll m$, in which case φ particles get trapped within the χ wall, forming a boson gas at temperature $T = 1/\beta$ with a thermal distribution of particle energies. In this case only particles with energies $E \geq m$ can escape from the wall to the vacuum outside. If $T \ll m$, i.e., $\beta m \gg 1$, then the number of particles $N(E) = (e^{\beta E} - 1)^{-1}$ with enough energy to escape is small, and the φ leakage rate is low, and the gas pressure tends to counteract the χ wall tension, allowing a spherical bubble to find equilibrium at a finite radius R. Bubbles of this scenario are dubbed "type 1" bubbles.

(2) $m_{\rm in} \gg m$, and in this case φ particles are expelled out of the χ wall and into surrounding vacuum. However, χ walls are themselves dynamic and can form closed bubbles through mechanisms mentioned earlier. If χ bubbles form before R-$\bar{\text{R}}$ annihilations complete, a φ gas will become trapped in the interior of the bubble, and exert an outward pressure tending to counteract the effects of bubble tension, allowing an equilibrium state for the bubble. Bubbles of this scenario are dubbed "type 2" bubbles.

III. TYPE 1 BUBBLE STABILITY

A. Bosonic Stabilization

For the type 1 bubble $m_{\rm in} \ll m$ and a gas of φ bosons is trapped inside the χ bubble wall, which we assume to take a spherical shape. Dynamical bubbles can emit radiation in the form of χ particles of mass $m_\chi = \sqrt{2\lambda}\eta$. Now consider a bubble which is shrinking under the influence of its wall tension. As it shrinks, the amount of bubble wall volume available to the φ gas decreases, and we expect the temperature of the gas to rise. We assume that the φ gas is relativistic, with effectively massless particles, so that a nonnegligible energy density ρ_G, number density n, and pressure p can exist to play a role in a stabilization of the bubble. (We ignore possible effects of χ particles emitted into the bubble's interior vacuum, as they can pass right through the χ wall - the reflection coefficient is zero[20] - and pass into the exterior vacuum. Similarly, we ignore the possibility of φ bosons inside of a closed ribbon loop exerting an outward pressure on the loop, since reflectionless scattering of φ particles from a ϕ wall is expected.) The radial force acting on the bubble wall is $F_R = -\partial\mathcal{E}/\partial R$,

where $\mathcal{E}$ is the configuration energy of the nontopological soliton composed of vacuum bubble and boson gas. If $F_R < 0$ the bubble shrinks, and for $F_R > 0$ the bubble tends to expand at the expense of φ particle energy, and at equilibrium the configuration energy $\mathcal{E}$ is minimized with $F_R = 0$.

The configuration energy $\mathcal{E}$ of the bubble is the sum of two terms, the energy $\mathcal{E}_W$ of the bubble wall due to wall surface energy density Σ (tension), and the energy $\mathcal{E}_G$ of the relativistic φ boson gas trapped within the wall, which has a volume of $w(4\pi R^2)$, where R is the bubble radius. Then

$$\mathcal{E} = \mathcal{E}_W + \mathcal{E}_G = \Sigma(4\pi R^2) + \rho_G(4\pi R^2 w) = 4\pi R^2 \left(\Sigma + \frac{\pi^2}{30} T^4 w \right) \tag{10}$$

where the φ gas energy density is $\rho_G = \frac{\pi^2}{30} T^4$. The equilibrium radius R is determined by minimizing $\mathcal{E}$ with respect to R. We must be careful, however, in this minimization. We want the minimal value of $\mathcal{E}$ for a given φ particle number N and a given entropy S. Therefore we consider a *virtual* variation of $\mathcal{E}$ holding N and S fixed. (One condition implies the other, since both are proportional to $T^3(4\pi R^2 w)$.) The number density of φ particles at temperature T is[21] $n = \frac{\zeta(3)}{\pi^2} T^3$ and the entropy density is[21] $s = \frac{2\pi^2}{45} T^3$. We then have

$$N = n(4\pi R^2 w) = \frac{4w\zeta(3)}{\pi} T^3 R^2, \qquad S = s(4\pi R^2 w) = \frac{8\pi^3 w}{45} T^3 R^2 \tag{11}$$

so that holding N and S constant during the virtual variation of $\mathcal{E}$ implies the constraint

$$T^3 R^2 = \frac{N\pi}{4w\zeta(3)} = \frac{45S}{8\pi^3 w} \equiv C^3; \qquad T = \frac{C}{R^{2/3}}; \qquad C = \left[\frac{N\pi}{4w\zeta(3)} \right]^{1/3} = \left[\frac{45S}{8\pi^3 w} \right]^{1/3} \tag{12}$$

Using the relation $T = CR^{-2/3}$ in (10) then gives

$$\frac{\mathcal{E}}{4\pi} = \Sigma R^2 + \frac{\pi^2 w C^4}{30} R^{-2/3} \tag{13}$$

Minimizing this expression by requiring $\partial\mathcal{E}/\partial R = 0$ results in an equilibrium radius given by

$$R^{8/3} = \frac{\pi^2 w C^4}{90\Sigma}; \qquad R = \left[\frac{\pi^2 w C^4}{90\Sigma} \right]^{3/8} \tag{14}$$

Equations (13) and (14) then give the bubble mass at equilibrium,

$$\frac{\mathcal{E}}{4\pi} = \frac{\mathcal{E}_W}{4\pi} + \frac{\mathcal{E}_G}{4\pi}; \quad \mathcal{E}_W = \Sigma R^2, \qquad \mathcal{E}_G = \frac{\pi^2 w C^4}{30} R^{-2/3} \tag{15}$$

Upon comparing the two energy terms we find, with the help of (14),

$$\frac{\mathcal{E}_G}{\mathcal{E}_W} = 3; \quad \mathcal{E} = \mathcal{E}_W + \mathcal{E}_G = 4\mathcal{E}_W = \frac{4}{3}\mathcal{E}_G = 16\pi\Sigma R^2 \tag{16}$$

so that a type 1 bubble of radius R at equilibrium has a mass $\mathcal{E} = 16\pi\Sigma R^2$. From (16) we also have $\mathcal{E}_G = 3\mathcal{E}_W$, which leads to [see, e.g., (10)] an equilibrium temperature given by

$$\frac{\pi^2}{30}T^4 w = 3\Sigma \quad \Longrightarrow \quad T = \left(\frac{90\Sigma}{\pi^2 w}\right)^{1/4} \tag{17}$$

Therefore, all type 1 bubbles equilibrate at this temperature, regardless of size.

B. Bubble Decay

Even with a bosonic stabilization mechanism, we don't expect the bubble to remain in equilibrium forever. This is because high energy φ particles with energies $E \geq m$ are energetically allowed to escape the bubble wall and eventually end up in the exterior vacuum. Although $T \ll m$ for a bubble near equilibrium, we expect a rate of leakage which is slow at first, but then increases due to an increase in temperature and average φ particle energy. A dynamical bubble can also shrink through emission of χ particles. Let us focus on a simple scenario of type 1 bubble decay due to φ particle loss. We make the simple assumption that each φ particle that escapes the bubble becomes a ϕ particle of mass m outside the bubble. Suppose the bubble is initially in a state near equilibrium with mass $\mathcal{E}_0$ and φ particle number N_0 at some initial time t_0. After a time $t - t_0$ the bubble has a mass $\mathcal{E}$ and φ particle number N, and has emitted $N_\phi = N_0 - N$ particles into the exterior vacuum. Then we can write $\mathcal{E}_0 = \mathcal{E} + N_\phi m + \mathcal{E}_X$, where $\mathcal{E}_X$ includes ϕ boson kinetic energy and χ radiation energy. The energy $\mathcal{E}_X$ will be a positive, monotonically increasing function of time t. Now let us rewrite this in the form

$$Nm - \mathcal{E} = K + \mathcal{E}_X \equiv Q(t) \tag{18}$$

where $K = N_0 m - \mathcal{E}_0$ and $Q(t)$ is a positive, monotonically increasing function. We see that $K > 0$ since the bubble is initially close to equilibrium and $N_0 m > \mathcal{E}_0$. To be more explicit, let us write

$$\frac{N}{4\pi R^2} = \frac{\zeta(3) w T^3}{\pi^2} \equiv DT^3, \qquad \frac{\mathcal{E}_G}{4\pi R^2} = \frac{\pi^2 w}{30}T^4 \equiv BT^4 \tag{19}$$

where

$$D = \frac{\zeta(3) w}{\pi^2}, \quad B = \frac{\pi^2 w}{30}, \quad \frac{B}{D} = 2.7, \quad \frac{D}{B} = .37 \tag{20}$$

As long as the bubble remains sufficiently close to equilibrium, with $T \ll m$, we can write (18) in the form

$$mDT^3 - BT^4 = \Sigma + \frac{Q}{4\pi R^2} \tag{21}$$

The first term on the left hand side dominates the second (and $K > 0$ for this same reason [22]), so that we have, approximately,

$$T \approx \left[\frac{1}{mD}\left(\Sigma + \frac{Q}{4\pi R^2}\right)\right]^{1/3} \tag{22}$$

for times where the bubble does not stray too far from equilibrium. As the bubble shrinks, the φ temperature increases, and the leakage rate increases.

Eventually, the bubble will have evolved too far away from an equilibrium state for the approximations above to remain valid. This is seen by noticing that at high enough temperatures the left hand side of (21) begins decreasing, while the right hand side is increasing. This happens for $T \gtrsim T_m$, where

$$T_m = \frac{3D}{4B} m \approx \frac{1}{4} m \tag{23}$$

locates the local maximum of the left hand side of (21). Upon approaching this temperature, we must assume that the bubble rapidly loses its φ gas, and undergoes an unchecked collapse. In fact, at the temperature T_m we have $\beta E \geq \beta m$ for particles energetic enough to escape the bubble wall, with $\beta m = m/T_m \approx 4$. The energy density of the portion of the gas with particles with energy $E \geq m$ is then given, approximately, by[23]

$$u(T_m) = \frac{T_m^4}{2\pi^2} \int_{\beta m=4}^{\infty} \frac{\eta^3}{e^\eta - 1} d\eta = \frac{T_m^4}{2\pi^2} I(4) \tag{24}$$

where $\eta = \beta E$ and $I(4) = 2.6$. The energy density of the entire boson gas is $\rho_G(T_m) = \frac{T_m^4}{2\pi^2} I(0)$ (taking the φ particles to be effectively massless, with $m_{\text{in}} \sim 0$), so that a comparison gives

$$\frac{u(T_m)}{\rho_G(T_m)} = \frac{I(4)}{I(0)} = \frac{2.6}{\pi^4/15} = .4 \tag{25}$$

So at $T \sim T_m$ roughly 40% of the bosonic gas has been lost, and the stabilization mechanism rapidly comes to an end. On the other hand, for $T \ll m$, i.e., $\beta m \gg 1$, we have

$$I(\beta m) = \int_{\beta m}^{\infty} \frac{\eta^3}{e^\eta - 1} d\eta \approx \int_{\beta m}^{\infty} \eta^3 e^{-\eta} d\eta \approx (\beta m)^3 e^{-\beta m} \ll 1 \tag{26}$$

so that $u(T)/\rho_G(T) \ll 1$ when the bubble is very close to equilibrium. Therefore the φ leakage rate is very low intially, but rapidly increases, so that the bosonic stabilization mechanism operates effectively for only a limited time span. After that, the type 1 bubble collapses.

IV. TYPE 2 BUBBLE STABILITY

A. Bosonic Stabilization

For the type 2 bubble $m_{\text{in}} \gg m$ and φ particles that are produced within the bubble wall are accelerated out of the wall into the vacuum as much lighter ϕ bosons. However, if the dynamical χ walls form bubbles while R-$\bar{\text{R}}$ annihilations are in progress, some of the ϕ particles will become trapped within the volume of the bubble's interior, and will produce

an outward pressure that has a stabilizing effect on the bubble. We assume the gas of light ϕ particles to be relativistic, with nonnegligible energy density ρ_G, number density n, and pressure p. Again, the radial force acting on the bubble wall is $F_R = -\partial\mathcal{E}/\partial R$, where $\mathcal{E}$ is the configuration energy of the nontopological soliton composed of the χ bubble wall and the enclosed ϕ gas. An equilibrium state exists when $\mathcal{E}$ is minimized for some radius R.

There are again two contributions to the energy $\mathcal{E}$, one from the bubble wall and one from the enclosed ϕ gas:

$$\mathcal{E} = \mathcal{E}_W + \mathcal{E}_G = \Sigma(4\pi R^2) + \rho_G\left(\frac{4}{3}\pi R^3\right) = 4\pi\left(\Sigma R^2 + \frac{\pi^2}{90}T^4 R^3\right) \tag{27}$$

where again $\rho_G = \frac{\pi^2}{30}T^4$. Again, we perform a virtual variation of $\mathcal{E}$ with respect to R while holding the ϕ particle number N and ϕ entropy S fixed. Again using $n = \frac{\zeta(3)}{\pi^2}T^3$ for the ϕ particle number density and $s = \frac{2\pi^2}{45}T^3$ for the ϕ entropy, we have

$$N = n\left(\frac{4}{3}\pi R^3\right) = \frac{4\zeta(3)}{3\pi}T^3R^3, \qquad S = s\left(\frac{4}{3}\pi R^3\right) = \frac{8\pi^3}{135}T^3R^3 \tag{28}$$

Requiring N and S to remain fixed during the virtual variation results in the constraint

$$T^3R^3 = \frac{3\pi N}{4\zeta(3)} = \frac{135 S}{8\pi^3} \equiv C_0^3; \quad T = \frac{C_0}{R}; \quad C_0 = \left[\frac{3\pi N}{4\zeta(3)}\right]^{1/3} = \left[\frac{135 S}{8\pi^3}\right]^{1/3} \tag{29}$$

Using the constraint $T = C_0/R$ in (27) results in

$$\frac{\mathcal{E}}{4\pi} = \Sigma R^2 + \frac{\pi^2 C_0^4}{90 R} \tag{30}$$

The equilibrium condition $\partial\mathcal{E}/\partial R = 0$ yields an equilibrium bubble radius

$$R = \left(\frac{\pi^2 C_0^4}{180\Sigma}\right)^{1/3} \equiv \left(\frac{C_1}{\Sigma}\right)^{1/3}; \quad C_1 = \frac{\pi^2 C_0^4}{180} \tag{31}$$

we can then write

$$\frac{\mathcal{E}_W}{4\pi} = C_1^{2/3}\Sigma^{1/3}, \quad \frac{\mathcal{E}_G}{4\pi} = 2C_1^{2/3}\Sigma^{1/3} = 2\frac{\mathcal{E}_W}{4\pi}, \quad \frac{\mathcal{E}_G}{\mathcal{E}_W} = 2 \tag{32}$$

Therefore, the mass of the type 2 bubble, when in equilibrium, is given by

$$\mathcal{E} = \mathcal{E}_W + \mathcal{E}_G = 3\mathcal{E}_W = \frac{3}{2}\mathcal{E}_G = 12\pi\Sigma R^2 \tag{33}$$

From $\mathcal{E}_G = 2\mathcal{E}_W$ it follows that at equilibrium the temperature of the ϕ gas is given by

$$\left(\frac{\pi^2}{30}T^4\right)\frac{4}{3}\pi R^3 = \Sigma\left(8\pi R^2\right) \implies T = \left(\frac{180\Sigma}{\pi^2 R}\right)^{1/4} \tag{34}$$

indicating that at equilibrium, larger bubbles have a lower temperature than smaller ones.

B. Bubble Decay

We would like to know under what conditions a type 2 bubble can remain in a long-lived near-equilibrium state. A long lived bubble must not leak ϕ particles to the outside vacuum at a significant rate, which requires that the reflection coefficient $\mathcal{R}$ of the bubble wall be near unity. For this condition to be met, the energies of most of the ϕ particles must satisfy[20] $E \ll E_c \equiv w^{-1}$, where we have introduced an energy $E_c = w^{-1}$, with w being the thickness of the bubble wall. For a typical ϕ particle energy $E \sim T$, (and therefore $\beta E \sim 1$) we then require $\beta E \ll \beta E_c$, i.e., $\beta E_c \gg 1$, to have $\mathcal{R} \approx 1$. Furthermore, under this condition the number of particles with such low energy will be $N(E) = (e^{\beta E} - 1)^{-1} \gg N(E_c)$, so that most ϕ particles will be reflected from the wall and remain inside the bubble. Another way to see this is to look at the fraction of the total energy density of the gas for particles having energies $E \geq E_c$. The energy density associated with this set of particle energies is given by[23]

$$u(T; \beta E_c) = \frac{T^4}{2\pi^2} \int_{\beta E_c}^{\infty} \frac{\eta^3 d\eta}{e^{\eta} - 1} = \frac{T^4}{2\pi^2} I(\beta E_c) \tag{35}$$

where $\eta = \beta E$. Now, for $\beta E_c \gg 1$ we have

$$I(\beta E_c) \approx \int_{\beta E_c}^{\infty} \eta^3 e^{-\eta} d\eta \approx (\beta E_c)^3 e^{-\beta E_c} \ll 1, \qquad (\beta E_c \gg 1) \tag{36}$$

The total energy density of the ϕ gas is $\rho_G = u(T;0) = \frac{T^4}{2\pi^2} I(0)$, where[23] $I(0) = \pi^4/15$. We therefore have

$$\frac{u(T; \beta E_c)}{\rho_G} = \frac{I(\beta E_c)}{I(0)} \approx \left(\frac{15}{\pi^4}\right) (\beta E_c)^3 e^{-\beta E_c} \ll 1, \qquad (\text{for } \beta E_c \gg 1) \tag{37}$$

Therefore, for the bosonic stabilization mechanism to be long-lived, with a very low ϕ leakage rate, we require $\beta E_c \gg 1$, i.e.,

$$T \ll w^{-1} \tag{38}$$

With a bubble remaining near equilibrium, this condition, along with (34) implies that the bubble radius R satisfy

$$R = \frac{180\Sigma}{\pi^2 T^4} \gg \Sigma w^4 \tag{39}$$

Therefore, larger equilibrium bubbles, with thin walls, will decay at a slower rate, and be longer lived than smaller ones.

V. SUMMARY

A model of two interacting real scalar fields χ and ϕ has been considered where the spontaneous breaking of a Z_2 symmetry associated with the field χ gives rise to domain walls, which are dynamical, and can interact to form closed bubbles. Within the core of a χ wall the ϕ field will tend to settle into ground states $\phi = \pm\phi_0$, breaking a Z_2 symmetry

associated with ϕ. However, the $+\phi_0$ and $-\phi_0$ domains will be uncorrelated beyond some coherence length ξ, and adjacent domains wil be separated by a domain "ribbon" (R) or "antiribbon" ($\bar{\mathrm{R}}$). These ribbon structures, confined to the core of the χ wall, can behave as cosmic strings would, and exist either as long wiggly objects or in the form of closed loops.

There will be R-$\bar{\mathrm{R}}$ annihilations taking place which result in the production of φ bosons, which are the elementary quanta of the $\pm\phi_0$ vacuua. If these φ bosons are produced within a bubble wall, they can get trapped within the wall, if $m_{\rm in} \ll m$, where $m_{\rm in}$ is the mass of the φ particles within the wall, where $\phi = \pm\phi_0 + \varphi$, and m is the ϕ particle mass in true vacuum outside the wall, where $\phi = 0$ locates the vacuum. These trapped φ bosons form a boson gas within the wall with pressure p, which tends to counteract the effects of the bubble wall tension Σ. On the other hand, if $m_{\rm in} \gg m$, the φ particles will be expelled from the wall, appearing as ϕ particles in vacuum. Some of these ϕ particles will get trapped within the bubble's enclosed volume, and unless a substantial fraction of these ϕ particles have a high enough energy to be transmitted through the wall, they will form a boson gas which exerts an outward pressure p, again, having a tendency to counteract the effects of the wall tension Σ. Bubbles of the former case, where $m_{\rm in} \ll m$, have been dubbed "type 1" bubbles, and bubbles of the latter case, where $m_{\rm in} \gg m$ are dubbed "type 2" bubbles. Conditions for a bosonic stabilization of each type of bubble due to a relativistic gas of bosonic particles have been determined.

It is found that type 1 bubbles can be metastable, finding an equilibrium radius where the effects of the boson gas pressure balance those of the wall tension, but this equilibrium exists for only a limited amount of time. This is due to the fact that φ particles with high enough energy, $E \geq m$, can escape from the wall, and eventually wind up in the exterior vacuum. The rate of leakage of boson particles is small initially, but as the bubble shrinks, the gas temperature rises, and the rate of φ particle loss rapidly increases, leading to an unchecked collapse of the type 1 bubbles.

Type 2 bubbles enclose a gas of ϕ bosons in the interior which exert an outward pressure on the bubble wall. Again, the bubble can stabilize with some radius R when the gas has a temperature T. While all type 1 bubbles equilibrate at the same temperature, regardless of size, a type 2 bubble equilibrates at a temperature $T \propto R^{-1/4}$, so that a larger bubble at equilibrium has a correspondingly lower temperature. Still, high energy ϕ particles can escape the bubble's interior by passing through the bubble wall, but the reflection coeffficient $\mathcal{R}$ is energy dependent, with $\mathcal{R}$ being large and the transmission coefficient $\mathcal{T}$ being small for low energies. Specifically, $\mathcal{R} \approx 1$ for particle energies $E \ll w^{-1}$, with w being the thickness of the bubble wall. For the bosonic stabilization mechanism to be long-lived, there should be only a negligible fraction of ϕ particles with high enough energy to escape, i.e., a very low leakage rate. The condition for this to occur is found to be given by $T \ll w^{-1}$, so that at equilibrium, the temperature is small in comparison to an energy $E_c \equiv w^{-1}$. For a bubble near equilibrium, this implies $R \gg \Sigma w^4$, so that larger equilibrium bubbles, or bubbles with very thin walls, may survive for longer periods of time before an eventual decay.

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[22] More specifically, we have $K = N_0 m - \mathcal{E}_0$ for a bubble near equilibrium, and $\mathcal{E}_0 \approx \frac{4}{3}\mathcal{E}_G$, so that $K > 0$ for $T \ll m$.

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